A logic of interrogation should be internalized in a modal logic for knowledge

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We propose a modal semantics of natural language questions that:

- takes advantage of the standardly available proof and model theory for modal logic;
- internalizes questions to combine the advantages of Groenendijk and Stokhof's intensional partition theory (1984; 1996; Groenendijk 1999) and Nelken and Francez's extensional bilattice theory (2000; 2002) of questions;
- encodes families of subquestions (Beck and Sharvit 2002; Sharvit and Beck 2001);
- distinguishes the exhaustivity of questions from the completeness of answers; and
- generalizes to conversations among multiple, overlapping groups of participants.

From knowing to asking. We follow the basic approach of Hintikka (1976) and Åqvist (1965), who interpret a question as a request for knowledge: 'bring it about that I know whether ...'. Focusing solely on the epistemic part of this request, the basic ingredients of our proposal are found in one's favorite modal logic for knowledge, perhaps S4 or S5; we only assume a normal modality, the T axiom (reflexive frames), and the Barcan formula both ways (constant domains). The necessity operator $\Box$ can be read as "it is known that" or "it is in the common ground that". Assertions are formulas of the form $\Box\phi$. For example, for it to be asserted that Alice is quitting is for it to be in the common ground that Alice is quitting: $\Box Qa$.

For any formula ϕ, we write $?\phi$ as shorthand for $\Box\phi \vee \Box\neg\phi$. Formulas of this form encode yes-no questions. For example, to know whether Alice is quitting is to either know that she is or know that she is not: $?Qa \equiv \Box Qa \vee \Box\neg Qa$. As for *wh*-questions, we encode them as formulas of the form $?\vec{x}.\,\phi$, which is shorthand for $\forall\vec{x}.\,?\phi$ (but see the refinements below), where $\vec{x}$ is one or more variables. For example, to know who is quitting is to know for each person whether he or she is quitting: $\forall x.\,?Qx$.

Our treatment of questions inherits well-developed proof and model theories from the modal logic it is couched in. In particular, it simultaneously features an intensional semantics and an extensional semantics, the latter provided that the modal depth of formulas (that is, the maximum number of $\Box$-nesting levels) is bounded. Thus our work formally relates much intuition that already underlies other treatments. The partition theory straightforwardly embeds into our theory. In fact, we formally prove that G&S's prized *answerhood* relation, predicting the relevance of an answer to a question, is exactly preserved. Moreover, the translation provides us with a sound and complete proof procedure for G&S's answerhood relation using techniques from Cerrito and Cialdea Mayer (2001). Our theory is also the first with an extensional semantics to predict that *is it either raining or not raining* ($?(R \vee \neg R)$) is a trivial question.

Internalized questions. As explained above, we let a question denote its *knowledge condition*: what it takes to know the complete answer. Knowledge conditions can further combine with other formulas, because our ? operator is *internalized*: it can apply not just at the top level of a formula or under some top-level universal quantifiers.

For example, we can not only conjoin questions (*do you have a license <u>and</u> who (else) has one?*: $?L \wedge ?x.\,Lx$) as G&S and N&F do, but also disjoin questions (*do you have a license <u>or</u> who (else) has one?*: $?L \vee ?x.\,Lx$) as N&F do without G&S's higher type-shifting. Also expressible are conditional questions (*if it's raining, may I borrow your umbrella*: $R \to ?U$). Such conditionals also significantly simplify the denotations of universal wide-scope readings of questions with quantifiers, as N&F do (*who recommends each candidate*: $\forall y.\,Cy \to ?x.\,Rxy$). We generate these denotations using standard techniques for quantifier scoping, whereas they are out of reach for G&S. Our theory is thus more expressive, while generalizing G&S's notion of entailment.

Plurality of questions. As explained above, we interpret a *wh*-question by universally quantifying over individual questions. This strategy has a strong connection to Beck and Sharvit's work on families of subquestions for explaining quantificational variability effects. It is tempting to view $?\vec{x}.\,\phi$ as an explicit encoding of the set of questions $\{\,?\phi[\vec{d}/\vec{x}] \mid \vec{d} \in D^n\,\}$, where D is the domain and x consists of n variables. As Beck and Sharvit show, there are cases in which the contextually salient set of subquestions may differ from that set. In such a case, we may allow the flexibility to interpret $?\vec{x}.\,\phi$ as the set of subquestions that is contextually salient.

Exhaustive questions versus complete answers. If an assertion ϕ entails a question ψ, then we say that ϕ is a *complete* answer to ψ. For example, asserting that it is raining and nobody is quitting ($\Box(R \wedge \forall x.\,\neg Qx)$) completely answers the question who is quitting ($?x.\,Qx$).

Completeness relates answers to questions; *exhaustivity* is a separate notion that applies to *wh*-questions only. The above encoding of *wh*-questions as formulas of the form $\forall\vec{x}.\,?\phi$ is *strongly exhaustive* in that it universally quantifies over $\vec{x}$. To know who is quitting in this sense is to know for each person x either that x is quitting or that x is not. By contrast, to know who is quitting in the *weakly exhaustive* sense is to know, for each person x who is quitting, that x is quitting: $\forall x.\,Qx \to \Box Qx$, or more generally $\forall\vec{x}.\,\phi \to \Box\phi$. Finally, a question like *where a gas station is*, in an appropriately desperate situation, is *non-exhaustive* and would be represented by existential quantification: $\exists x.\,\Box S\,x$. Any assertion of a gas station's location qualifies as a complete answer.

This distinct representation for non-exhaustive questions reflects the fact that they cannot be interpreted as a family of subquestions. For example, if Alice knows the exact location of even a single gas station, then Alice knows exactly where to get gas. If Alice runs out of gas while driving, and she asks Bob whether he knows where to get gas, it would be odd for Bob to answer *with few exceptions, I do*, even if Bob knows the exact location of all but a few gas stations in the area. This oddity is because no (countable) set of subquestions is contextually salient.

Multi-party conversations. Because the partition theory of questions embeds into our semantics, we can recast Groenendijk's game of interrogation (1999) in our terms. We can then extend the game from one interrogator and one witness to multiple, overlapping groups of participants. Keeping track of such a game requires maintaining the knowledge state of each agent or group of agents. For example, if we fix a single interrogator but allow witnesses to enter and leave the room, then the inquisitiveness requirement permits the interrogator to ask the same question again after a witness enters, but not leaves, the room.

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